3| Probability Topics

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Introduction

Figure 3.1 Meteor showers are rare, but the probability of them occurring can be calculated. (credit: Navicore/flickr)

It is often necessary to "guess" about the outcome of an event in order to make a decision. Politicians study polls to guess their likelihood of winning an election. Teachers choose a particular course of study based on what they think students can comprehend. Doctors choose the treatments needed for various diseases based on their assessment of likely results. You may have visited a casino where people play games chosen because of the belief that the likelihood of winning is good. You may have chosen your course of study based on the probable availability of jobs.

You have, more than likely, used probability. In fact, you probably have an intuitive sense of probability. Probability deals with the chance of an event occurring. Whenever you weigh the odds of whether or not to do your homework or to study for an exam, you are using probability. In this chapter, you will learn how to solve probability problems using a systematic approach.

3.1 | Terminology

Probability is a measure that is associated with how certain we are of outcomes of a particular experiment or activity. An **experiment** is a planned operation carried out under controlled conditions. If the result is not predetermined, then the experiment is said to be a **chance** experiment. Flipping one fair coin twice is an example of an experiment.

A result of an experiment is called an **outcome**. The **sample space** of an experiment is the set of all possible outcomes. Three ways to represent a sample space are: to list the possible outcomes, to create a tree diagram, or to create a Venn diagram. The uppercase letter *S* is used to denote the sample space. For example, if you flip one fair coin, *S* = {*H*, *T*} where *H* = heads and *T* = tails are the outcomes.

An **event** is any combination of outcomes. Upper case letters like *A* and *B* represent events. For example, if the experiment is to flip one fair coin, event *A* might be getting at most one head. The probability of an event *A* is written *P*(*A*).

The **probability** of any outcome is the **long-term relative frequency** of that outcome. **Probabilities are between zero and one, inclusive** (that is, zero and one and all numbers between these values). *P*(*A*) = 0 means the event *A* can never happen. *P*(*A*) = 1 means the event *A* always happens. *P*(*A*) = 0.5 means the event *A* is equally likely to occur or not to occur. For example, if you flip one fair coin repeatedly (from 20 to 2,000 to 20,000 times) the relative frequency of heads approaches 0.5 (the probability of heads).

Equally likely means that each outcome of an experiment occurs with equal probability. For example, if you toss a **fair**, six-sided die, each face (1, 2, 3, 4, 5, or 6) is as likely to occur as any other face. If you toss a fair coin, a Head (*H*) and a Tail (*T*) are equally likely to occur. If you randomly guess the answer to a true/false question on an exam, you are equally likely to select a correct answer or an incorrect answer.

To calculate the probability of an event A when all outcomes in the sample space are equally likely, count the number of outcomes for event A and divide by the total number of outcomes in the sample space. As an equation this is:

$$P(A) = \frac{number\ of\ ways\ to\ get\ A}{Total\ number\ of\ possible\ outcomes}$$

For example, if you toss a fair dime and a fair nickel, the sample space is {*HH*, *TH*, *HT*, *TT*} where *T* = tails and *H* = heads. The sample space has four outcomes. *A* = getting one head. There are two outcomes that meet this condition {*HT*, *TH*}, so $P(A) = \frac{2}{4} = 0.5$.

Suppose you roll one fair six-sided die, with the numbers {1, 2, 3, 4, 5, 6} on its faces. Let event *E* = rolling a number that is at least five. There are two outcomes {5, 6}. $P(E) = \frac{2}{6}$. . If you were to roll the die only a few times, you would not be surprised if your observed results did not match the probability. If you were to roll the die a very large number of times, you would expect that, overall, $\frac{2}{6}$ of the rolls would result in an outcome of "at least five". You would not expect exactly $\frac{2}{6}$. The long-term relative frequency of obtaining this result would approach the theoretical probability of $\frac{2}{6}$ as the number of repetitions grows larger and larger.

This important characteristic of probability experiments is known as the **law of large numbers** which states that as the number of repetitions of an experiment is increased, the relative frequency obtained in the experiment tends to become closer and closer to the theoretical probability. Even though the outcomes do not happen according to any set pattern or order, overall, the long-term observed relative frequency will approach the theoretical probability. (The word **empirical** is often used instead of the word observed.)

It is important to realize that in many situations, the outcomes are not equally likely. A coin or die may be **unfair**, or **biased**. Two math professors in Europe had their statistics students test the Belgian one Euro coin and discovered that in 250 trials, a head was obtained 56% of the time and a tail was obtained 44% of the time. The data seem to show that the coin is not a fair coin; more repetitions would be helpful to draw a more accurate conclusion about such bias. Some dice may be biased. Look at the dice in a game you have at home; the spots on each face are usually small holes carved out and then painted to make the spots visible. Your dice may or may not be biased; it is possible that the outcomes may be affected by the slight weight differences due to the different numbers of holes in the faces. Gambling casinos make a lot of money depending on outcomes from rolling dice, so casino dice are made differently to eliminate bias. Casino dice have flat faces; the holes are completely filled with paint having the same density as the material that the dice are made out of so that each face is equally likely to occur. Later we will learn techniques to use to work with probabilities for events that are not equally likely.

"∪" Event: The Union

An outcome is in the event $A \cup B$ if the outcome is in A or is in B or is in both A and B. For example, let A = {1, 2, 3, 4, 5} and B = {4, 5, 6, 7, 8}. $A \cup B$ = {1, 2, 3, 4, 5, 6, 7, 8}. Notice that 4 and 5 are NOT listed twice.

" ∩ " Event: The Intersection

An outcome is in the event $A \cap B$ if the outcome is in both *A* and *B* at the same time. For example, let *A* and *B* be {1, 2, 3, 4, 5} and {4, 5, 6, 7, 8}, respectively. Then $A \cap B$ = {4, 5}.

The **complement** of event *A* is denoted *A′* (read "*A* prime"). *A′* consists of all outcomes that are **NOT** in *A*. Notice that *P*(*A*)+ *P*(*A′*) = 1. For example, let *S* = {1, 2, 3, 4, 5, 6} and let *A* = {1, 2, 3, 4}. Then *A′* = {5, 6}.

$P(A) = \frac{4}{6}$, $P(A') = \frac{2}{6}$ and $P(A) + P(A') = \frac{4}{6} + \frac{2}{6} = 1$.

The **conditional probability** of *A* given *B* is written $P(A \mid B)$. $P(A \mid B)$ is the probability that event *A* will occur given that the event *B* has already occurred. **A conditional reduces the sample space**. We calculate the probability of *A* from the reduced sample space *B*. The formula to calculate $P(A|B)$ is $P(A|B) = \frac{P(A \cap B)}{P(B)}$ where $P(B)$ is greater than zero.

For example, suppose we toss one fair, six-sided die. The sample space *S* = {1, 2, 3, 4, 5, 6}. Let *A* = face is 2 or 3 and *B* = face is even (2, 4, 6). To calculate *P*(*A* | *B*), we count the number of outcomes 2 or 3 in the sample space *B* = {2, 4, 6}. Then we divide that by the number of outcomes *B* (rather than *S*).

We get the same result by using the formula. Remember that *S* has six outcomes.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{\text{the number of outcomes that are 2 or 3 and even in } S}{6}}{\frac{\text{the number of outcomes that are even in } S}{6}} = \frac{1/6}{3/6} = \frac{1}{3}$$

Odds

The odds of an event presents the probability as a ratio of success to failure. This is common in various gambling formats. Mathematically, the odds of an event can be defined as:

$$\frac{P(A)}{1 - P(A)}$$

where *P*(*A*) is the probability of success and of course 1 − *P*(*A*) is the probability of failure. Odds are always quoted as "numerator to denominator," e.g. 2 to 1. Here the probability of winning is twice that of losing; thus, the probability of winning is 0.66. A probability of winning of

0.60 would generate odds in favor of winning of 3 to 2. While the calculation of odds can be useful in gambling venues in determining payoff amounts, it is not helpful for understanding probability or statistical theory.

Understanding Terminology and Symbols

It is important to read each problem carefully to think about and understand what the events are. Understanding the wording is the first very important step in solving probability problems. Reread the problem several times if necessary. Clearly identify the event of interest. Determine whether there is a condition stated in the wording that would indicate that the probability is conditional; carefully identify the condition, if any.

Example 3-1

The sample space *S* is the whole numbers starting at one and less than 20.

a) *S* = ________________________________

Let event *A* = the even numbers and event *B* = numbers greater than 13.

b) *A* = _______, *B* = _______

c) *P(A)* = _______, *P(B)* = ________

d) *A* ∩ *B* =_______, *A OR B* = __________

e) *P(A* ∩ *B)* = _______, *P(A* ∪ *B)* = _______

f) *A′* =_______, *P(A′)* =________

g) *P(A)* + *P(A′)* = ________

h) *P(A* | *B)* = _____, *P(B* | *A)* = ______; *are the probabilities equal?*

Solution 3-1

a) S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}

b) A = {2, 4, 6, 8, 10, 12, 14, 16, 18}, B = {14, 15, 16, 17, 18, 19}

c) $P(A) = \frac{9}{19}, P(B) = \frac{6}{19}$

d) $A \cap B = \{14, 16, 18\}, A\ OR\ B = \{2, 4, 6, 8, 10, 12, 14, 16, 17, 18, 19\}$

e) $P(A \cap B) = \frac{3}{19},\ P(A \cup B) = \frac{12}{19}$

f) $A' = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\},\ P(A') = \frac{10}{19}$

g) $P(A) + P(A') = \frac{9}{19} + \frac{10}{19} = 1$

h) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3}{6}$, $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{3}{9}$, No, the probabilities are not equal.

Example 3-2

A fair, six-sided die is rolled. Describe the sample space *S*, identify each of the following events with a subset of *S* and compute its probability (an outcome is the number of dots that show up).

a) Event *T* = the outcome is two.

b) Event *A* = the outcome is an even number.

c) Event *B* = the outcome is less than four.

d) The complement of *A*.

e) *A* | *B*

f) *B* | *A*

g) *A* ∩ *B*

h) *A* ∪ *B*

i) *A* ∪ *B′*

j) Event *N* = the outcome is a prime number.

k) Event I = the outcome is seven.

Solution 3-2

a) $T = \{2\}, P\{T\} = \frac{1}{6}$

b) $A = \{2, 4, 6\}, P(A) = \frac{1}{2}$

c) $B = \{1, 2, 3\}, P(B) = \frac{1}{2}$

d) $A' = \{1, 3, 5\},\ P(A') = \frac{1}{2}$

e) $A|B = \{2\},\ P(A|B) = \frac{1}{3}$

f) $B|A = \{2\},\ P(B|A) = \frac{1}{3}$

g) $A \cap B = \{2\}, P(A \cap B) = \frac{1}{6}$

h) $A \cup B = \{1, 2, 3, 4, 5, 6\}, P(A \cup B) = \frac{5}{6}$

i) $A \cup B' = \{2, 4, 5, 6\}, P(A \cup B') = \frac{2}{3}$

j) $N = \{2, 3, 5\}, P(N) = \frac{1}{2}$

k) A six-sided die does not have seven dots. $P(7) = 0$

Example 3-3

Table 3-1 describes the distribution of a random sample *S* of 100 individuals, organized by gender and whether they are right- or left-handed.

	Right-handed	**Left-handed**
Males	43	9
Females	44	4

Table 3-1

Let's denote the events *M* = the subject is male, *F* = the subject is female, *R* = the subject is right-handed, *L* = the subject is left-handed. Compute the following probabilities:

a) $P(M)$

b) $P(F)$

c) $P(R)$

d) $P(L)$

e) $P(M \cap R)$

f) $P(F \cap L)$

g) $P(M \cup F)$

h) $P(M \cup R)$

i) $P(F \cup L)$

j) $P(M')$

k) $P(R \mid M)$

l) $P(F \mid L)$

m) $P(L \mid F)$

Solution 3-3

a) $P(M) = 0.52$

b) $P(F) = 0.48$

c) $P(R) = 0.87$

d) $P(L) = 0.13$

e) $P(M \cap R) = 0.43$

f) $P(F \cap L) = 0.04$

g) $P(M \cup F) = 1$

h) $P(M \cup R) = 0.96$

i) $P(F \cup L) = 0.57$

j) $P(M') = 0.48$

k) $P(R \mid M) = 0.8269$ (rounded to four decimal places)

l) $P(F \mid L) = 0.3077$ (rounded to four decimal places)

m) $P(L \mid F) = 0.0833$

3.2 | Independent and Mutually Exclusive Events

Independent and mutually exclusive do **not** mean the same thing.

Independent Events

Two events are independent if one of the following are true:

- $P(A|B) = P(A)$
- $P(B|A) = P(B)$
- $P(A \cap B) = P(A)P(B)$

Two events *A* and *B* are **independent** if the knowledge that one occurred does not affect the chance the other occurs. For example, the outcomes of two roles of a fair die are independent events. The outcome of the first roll does not change the probability for the outcome of the second roll. To show two events are independent, you must show **only one** of the above conditions. If two events are NOT independent, then we say that they are **dependent**.

Sampling may be done **with replacement** or **without replacement**.

- **With replacement**: If each member of a population is replaced after it is picked, then that member has the possibility of being chosen more than once. When sampling is done with replacement, then events are considered to be independent, meaning the result of the first pick will not change the probabilities for the second pick.
- **Without replacement**: When sampling is done without replacement, each member of a population may be chosen only once. In this case, the probabilities for the second pick are affected by the result of the first pick. The events are considered to be dependent or not independent.

If it is not known whether *A* and *B* are independent or dependent, **assume they are dependent until you can show otherwise**.

Example 3-4

You have a fair, well-shuffled deck of 52 cards. It consists of four suits. The suits are clubs, diamonds, hearts and spades. There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *J* (jack), *Q* (queen), *K* (king) of that suit. Describe a possible scenario of picking three cards:

a) with replacement.

b) without replacement.

Solution 3-4

a) Sampling with replacement. Suppose you pick three cards with replacement. The first card you pick out of the 52 cards is the *Q* of spades. You put this card back, reshuffle the cards and pick a second card from the 52-card deck. It is the ten of clubs. You put this card back, reshuffle the cards and pick a third card from the 52-card deck. This time, the card is the *Q* of spades again. Your picks are {*Q* of spades, ten of clubs, *Q* of spades}. You have picked the *Q* of spades twice. You pick each card from the 52-card deck.

b) Sampling without replacement. Suppose you pick three cards without replacement. The first card you pick out of the 52 cards is the *K* of hearts. You put this card aside and pick the second card from the 51 cards remaining in the deck. It is the three of diamonds. You put this card aside and pick the third card from the remaining 50 cards in the deck. The third card is the *J* of spades. Your picks are {*K* of hearts, three of diamonds, *J* of spades}. Because you have picked the cards without replacement, you cannot pick the same card twice. The probability of picking the three of diamonds is called a conditional probability because it is conditioned on what was picked first. This is true also of the probability of picking the J of spades. The probability of picking the J of spades is actually conditioned on *both* the previous picks.

Try It 3-1

You have a fair, well-shuffled deck of 52 cards. It consists of four suits. The suits are clubs, diamonds, hearts and spades. There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *J* (jack), *Q* (queen), *K* (king) of that suit. Three cards are picked at random.

a) Suppose you know that the picked cards are *Q* of spades, *K* of hearts and *Q* of spades. Can you decide if the sampling was with or without replacement?

b) Suppose you know that the picked cards are *Q* of spades, *K* of hearts, and *J* of spades. Can you decide if the sampling was with or without replacement?

Chapter 3 Try It Solutions

Example 3-5

You have a fair, well-shuffled deck of 52 cards. It consists of four suits. The suits are clubs, diamonds, hearts, and spades. There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *J* (jack), *Q* (queen), and *K* (king) of that suit. *S* = spades, *H* = Hearts, *D* = Diamonds, *C* = Clubs.

a) Suppose you pick four cards, but do not put any cards back into the deck. Your cards are QS, 1D, 1C, QD.

b) Suppose you pick four cards and put each card back before you pick the next card. Your cards are KH, 7D, 6D, KH.

Which of a) or b) did you sample with replacement and which did you sample without replacement?

Solution 3-5

a) Without replacement; b) With replacement

Try It 3-2

You have a fair, well-shuffled deck of 52 cards. It consists of four suits. The suits are clubs, diamonds, hearts, and spades. There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *J* (jack), *Q* (queen), and *K* (king) of that suit. *S* = spades, *H* = Hearts, *D* = Diamonds, *C* = Clubs. Suppose that you sample four cards without replacement. Which of the following outcomes are possible? Answer the same question for sampling with replacement.

a) *QS*, 1*D*, 1*C*, *QD*

b) *KH*, 7*D*, 6*D*, *KH*

Chapter 3 Try It Solutions

Mutually Exclusive Events

A and *B* are **mutually exclusive** events if they cannot occur at the same time. Said another way, if *A* occurred then *B* cannot occur and vise-a-versa. This means that *A* and *B* do not share any outcomes and $P(A \cap B)=0$P(A∩B)=0.

For example, suppose the sample space *S* = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Let *A* = {1, 2, 3, 4, 5}, *B* = {4, 5, 6, 7, 8}, and *C* = {7, 9}. A ∩ B = {4, 5}. P(A∩B) = $\frac{2}{10}$ and is not equal to zero. Therefore, *A* and *B* are not mutually exclusive. *A* and *C* do not have any numbers in common so P(A∩C)=0. Therefore, *A* and *C* are mutually exclusive.

If it is not known whether *A* and *B* are mutually exclusive, **assume they are not until you can show otherwise**. The following examples illustrate these definitions and terms.

Example 3-6

Flip two fair coins. (This is an experiment.) The sample space is {*HH*, *HT*, *TH*, *TT*} where *T* = tails and *H* = heads. The outcomes are *HH*, *HT*, *TH*, and *TT*. The outcomes HT and TH are different. The *HT* means that the first coin showed heads and the second coin showed tails. The *TH* means that the first coin showed tails and the second coin showed heads.

a) Let *A* = the event of getting **at most one tail**. (At most one tail means zero or one tail.) Then *A* can be written as {*HH*, *HT*, *TH*}. The outcome *HH* shows zero tails. *HT* and *TH* each show one tail.

   Let *B* = the event of getting all tails. *B* can be written as {*TT*}. *B* is the **complement** of *A*, so *B* = *A′*. Also, *P*(*A*) + *P*(*B*) = *P*(*A*) + *P*(*A′*) = 1.

What are the probabilities of A and B?

b) Let *C* = the event of getting all heads. *C* = $\{HH\}$. What is $P(B)$?

c) Let *D* = event of getting **more than one** tail. *D* = $\{TT\}$. What is $P(D)$?

d) Let *E* = event of getting a head on the first roll. (This implies you can get either a head or tail on the second roll.) *E* = $\{HT, HH\}$. What is $P(E)$?

e) Find the probability of getting **at least one** (one or two) tail in two flips. Let *F* = event of getting at least one tail in two flips. What is $P(F)$?

Solution 3-6

a) The probabilities for *A* and for *B* are $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{4}$.

b) Since *B* = $\{TT\}$, $\mathrm{P}(B \cap C) = 0$. *B* and *C* are mutually exclusive. (*B* and *C* have no members in common because you cannot have all tails and all heads at the same time.)

c) $P(D) = \frac{1}{4}$

d) $P(E) = \frac{2}{4}$

e) *F* = $\{HT, TH, TT\}$. $P(F) = \frac{3}{4}$

Try It 3-3

Draw two cards from a standard 52-card deck with replacement. Find the probability of getting at least one black card.

Chapter 3 Try It Solutions

Example 3-7

Flip two fair coins. Find the probabilities of the events.

a) Let F = the event of getting at most one tail (zero or one tail).

b) Let G = the event of getting two faces that are the same.

c) Let H = the event of getting a head on the first flip followed by a head or tail on the second flip.

d) Are F and G mutually exclusive?

e) Let J = the event of getting all tails. Are J and H mutually exclusive?

Solution 3-7

Look at the sample space in Example 3-6.

a) Zero (0) or one (1) tails occur when the outcomes *HH*, *TH*, *HT* show up. $P(F) = \frac{3}{4}$

b) Two faces are the same if *HH* or *TT* show up. $P(G) = \frac{2}{4}$

c) A head on the first flip followed by a head or tail on the second flip occurs when *HH* or *HT* show up. $P(H) = \frac{2}{4}$

d) *F* and *G* share *HH* so P(F∩G) is not equal to zero (0). *F* and *G* are not mutually exclusive.

e) Getting all tails occurs when tails shows up on both coins (*TT*). *H*'s outcomes are *HH* and *HT*.

f) *J* and *H* have nothing in common so P(J∩H) = 0. *J* and *H* are mutually exclusive.

Try It 3-4

A box has two balls, one white and one red. We select one ball, put it back in the box, and select a second ball (sampling with replacement). Find the probability of the following events:

a) Let *F* = the event of getting the white ball twice.

b) Let *G* = the event of getting two balls of different colors.

c) Let *H* = the event of getting white on the first pick.

d) Are *F* and *G* mutually exclusive?

e) Are *G* and *H* mutually exclusive?

Chapter 3 Try It Solutions

Example 3-8

Roll one fair, six-sided die. The sample space is {1, 2, 3, 4, 5, 6}. Let event *A* = a face is odd. Then *A* = {1, 3, 5}. Let event *B* = a face is even. Then *B* = {2, 4, 6}.

Find the complement of A, A′. The complement of A, A′, is B because A and B together make up the sample space. P(A) + P(B) = P(A) + P(A′) = 1. Also, $P(A) = \frac{3}{6}$ and $P(B) = \frac{3}{6}$.

Let event C = odd faces larger than two. Then C = {3, 5}. Let event D = all even faces smaller than five. Then D = {2, 4}. P(C∩D)=0 because you cannot have an odd and even face at the same time.
Therefore, C and D are mutually exclusive events.

Let event E = all faces less than five. E = {1, 2, 3, 4}

a) Are C and E mutually exclusive events? (Answer yes or no.) Why or why not?

b) Find $P(C|A)$.

Solution 3-8

a) No. C = {3, 5} and E = {1, 2, 3, 4}. $C \cap E = \{3\}$ and $P(C \cap E) = \frac{1}{6}$. To be mutually exclusive, P(C∩E) must be zero.

b) To find P(C|A), find the probability of C using the sample space A. You have reduced the sample space from the original sample space {1, 2, 3, 4, 5, 6} to {1, 3, 5}. So, $P(C|A) = \frac{2}{3}$.

Try It 3-5

Let event *A* = learning Spanish. Let event *B* = learning German. Then $A \cap B$ = learning Spanish and German. Suppose $P(A) = 0.4$ and $P(B) = 0.2$. $P(A \cap B) = 0.08$. Are events *A* and *B* independent? Hint: You must show ONE of the following:

$P(A|B) = P(A)$

$P(B|A) = P(B)$

$P(A \cap B) = P(A)P(B)$

Chapter 3 Try It Solutions

Example 3-9

Let event *G* = taking a math class. Let event *H* = taking a science class. Then, G ∩ H = taking a math class and a science class.
Suppose P(G)=0.6,P(H)=0.5, and P(G∩H)=0.3. Are *G* and *H* independent?

If *G* and *H* are independent, then you must show **ONE** of the following:

- $P(G|H)=P(G)$
- $P(H|G)=P(H)$
- $P(G \cap H)=P(G)P(H)$

NOTE: The choice you make depends on the information you have. You could choose any of the methods here because you have the necessary information.

a) Show that $P(G|H)=P(G)$.

b) Show $P(G \cap H)=P(G)P(H)$.

Solution 3-9

a) $P(G|H) = \frac{P(G \cap H)}{P(H)} = \frac{0.3}{0.5} = 0.6 = P(G)$

b) $P(G)P(H) = (0.6)(0.5) = 0.3 = P(G \cap H)$

Try It 3-6

In a bag, there are six red marbles and four green marbles. The red marbles are marked with the numbers 1, 2, 3, 4, 5, and 6. The green marbles are marked with the numbers 1, 2, 3, and 4.

R = a red marble

G = a green marble

O = an odd-numbered marble

The sample space is $S = \{R1, R2, R3, R4, R5, R6, G1, G2, G3, G4\}$.

S has ten outcomes. What is $P(G \cap O)$?

Chapter 3 Try It Solutions

Example 3-10

Let event *C* = taking an English class. Let event *D* = taking a speech class.

Suppose $P(C)=0.75$, $P(D)=0.3$, $P(C|D)=0.75$ and $P(C \cap D)=0.225$.

Justify your answers to the following questions numerically.

a) Are *C* and *D* independent?

b) Are *C* and *D* mutually exclusive?

c) What is P(D|C)?

Solution 3-10

a) Yes, because P(C|D)=P(C).

b) No, because P(C∩D) is not equal to zero.

c) $P(D \mid C) = \frac{P(C \cap D)}{P(C)} = \frac{0.225}{0.75} = 0.3$

Try It 3-7

A student goes to the library. Let events B = the student checks out a book and D = the student checks out a DVD. Suppose that P(B)=0.40, P(D)=0.30 and P(B∩D)=0.20.

a) Find P(B|D)P(B|D).

b) Find P(D|B)P(D|B).

c) Are B and D independent?

d) Are B and D mutually exclusive?

Chapter 3 Try It Solutions

Example 3-11

In a box there are three red cards and five blue cards. The red cards are marked with the numbers 1, 2, and 3, and the blue cards are marked with the numbers 1, 2, 3, 4, and 5. The cards are well-shuffled. You reach into the box (you cannot see into it) and draw one card.

Let R = red card is drawn, B = blue card is drawn, E = even-numbered card is drawn.

The sample space S = R1, R2, R3, B1, B2, B3, B4, B5. S has eight outcomes.

$P(R) = \frac{3}{8}. P(B) = \frac{5}{8}.$ P(R∩B)=0. (You cannot draw one card that is both red and blue.)

$P(E) = \frac{3}{8}$. (There are three even-numbered cards, R2, B2, and B4.)

a) What is $P(E|B)$? What is $P(B|E)$?

b) Are R and B mutually exclusive?

c) Let G = card with a number greater than 3. G = {B4, B5}. $\boldsymbol{P}(G) = \frac{2}{8}$.
Let H = blue card numbered between one and four, inclusive. H = {B1, B2, B3, B4}. What is $P(G|H)$? Are G and H independent?

Solution 3-11

a) $P(E \mid B) = \frac{2}{5}$. (There are five blue cards: B1, B2, B3, B4, and B5. Out of the blue cards, there are two even cards; B2 and B4.)

$P(B|E) = \frac{2}{3}$. (There are three even-numbered cards: R2, B2, and B4. Out of the even-numbered cards, two are blue; B2 and B4.)

b) The events R and B are mutually exclusive because P(R∩B)=0.

c) $P(G \mid H) = \frac{1}{4}$. (The only card in H that has a number greater than three is B4.) Since $\frac{2}{8} = \frac{1}{4}$, P(G)=P(G|H), which means that G and H are independent.

Try It 3-8

In a basketball arena,

- 70% of the fans are rooting for the home team.
- 25% of the fans are wearing blue.
- 20% of the fans are wearing blue and are rooting for the away team.

Of the fans rooting for the away team, 67% are wearing blue.

Let *A* be the event that a fan is rooting for the away team.
Let *B* be the event that a fan is wearing blue.
Are the events of rooting for the away team and wearing blue independent? Are they mutually exclusive?

Chapter 3 Try It Solutions

Example 3-12

In a particular college class, 60% of the students are female. Fifty percent of all students in the class have long hair. Forty-five percent of the students are female and have long hair. Of the female students, 75% have long hair. Let *F* be the event that a student is female. Let *L* be the event that a student has long hair. One student is picked randomly. Are the events of being female and having long hair independent?

The following probabilities are given in this example:

- $P(F)=0.60$; $P(L)=0.50$
- $P(F \cap L)=0.45$
- $P(L|F)=0.75$

NOTE

The choice you make depends on the information you have. You could use the first or last condition on the list for this example. You do not know $P(F|L)$ yet, so you cannot use the second condition.

Solution 3-12

Solution 1

Check whether $P(F \cap L)=P(F)$. We are given that $P(F \cap L)=0.45$, but $P(F)P(L)=(0.60)(0.50)=0.30$. The events of being female and having long hair are not independent because $P(F \cap L)$ does not equal $P(F)P(L)$.

Solution 2

Check whether $P(L|F)$ equals $P(L)$. We are given that $P(L|F)=0.75$, but $P(L)=0.50$; they are not equal. The events of being female and having long hair are not independent.

Interpretation of Results

The events of being female and having long hair are not independent; knowing that a student is female changes the probability that a student has long hair.

Try It 3-9

Mark is deciding which route to take to work. His choices are I = the Interstate and F = Fifth Street.

- $P(I)=0.44$ and $P(F)=0.56$
- $P(I \cap F)=0$ because Mark will take only one route to work.

What is the probability of $P(I \cup F)$?

Chapter 3 Try It Solutions

Example 3-13

a) Toss one fair coin (the coin has two sides, H and T). The outcomes are _________. Count the outcomes. There are _____ outcomes.

b) Toss one fair, six-sided die (the die has 1, 2, 3, 4, 5 or 6 dots on a side). The outcomes are __________________. Count the outcomes. There are ___ outcomes.

c) Multiply the two numbers of outcomes. The answer is ________.

d) If you flip one fair coin and follow it with the toss of one fair, six-sided die, the answer to c is the number of outcomes (size of the sample space). What are the outcomes? (Hint: Two of the outcomes are H1 and T6.)

e) Event A = heads (H) on the coin followed by an even number (2, 4, 6) on the die.
A = {___________________}. Find P(A).

f) Event B = heads on the coin followed by a three on the die. B = {_________}. Find P(B).

g) Are A and B mutually exclusive? (Hint: What is P(A∩B)? If P(A∩B)=0, then A and B are mutually exclusive.)

h) Are A and B independent? (Hint: Is P(A∩B)=P(A)P(B)?
If P(A∩B)=P(A)P(B), then A and B are independent. If not, then they are dependent).

Solution 3-13

a) *H* and *T*; 2

b) 1, 2, 3, 4, 5, 6; 6

c) 2(6) = 12

d) *T*1, *T*2, *T*3, *T*4, *T*5, *T*6, *H*1, *H*2, *H*3, *H*4, *H*5, *H*6

e) *A* = {*H*2, *H*4, *H*6}; $P(A) = \frac{3}{12}$

f) *B* = {*H*3}; $P(B) = \frac{1}{12}$

g) Yes, because P(A∩B)=0

h) P(A∩B)=0. $P(A)P(B) = \frac{3}{12}$. P(A∩B) does not equal P(A)P(B), so A and B are dependent.

Try It 3-10

A box has two balls, one white and one red. We select one ball, put it back in the box, and select a second ball (sampling with replacement). Let *T* be the event of getting the white ball twice, *F* the event of picking the white ball first, *S* the event of picking the white ball in the second drawing.

a) Compute P(T).

b) Compute P(T|F).

c) Are *T* and *F* independent?.

d) Are *F* and *S* mutually exclusive?

e) Are *F* and *S* independent?

Chapter 3 Try It Solutions

3.3 | Two Basic Rules of Probability

When calculating probability, there are two rules to consider when determining if two events are independent or dependent and if they are mutually exclusive or not.

The Multiplication Rule

If *A* and *B* are two events defined on a **sample space**, then:

$$P(A \cap B) = P(B)P(A|B)$$

We can think of the intersection symbol as substituting for the word "and".

This rule may also be written as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

This equation is read as the probability of *A* given *B* equals the probability of *A* and *B* divided by the probability of *B*.

Multiplication Rule for Independent Events

If *A* and *B* are **independent** events, then P(A|B)=P(A). Therefore, for **independent events**, the multiplication rule can be written as:

$$P(A \cap B) = P(A)P(B)$$

NOTE

One easy way to remember the multiplication rule is that the word "and" means that the event has to satisfy two conditions. For example, the name drawn from the class roster is to be both a female and a sophomore. It is harder to satisfy two conditions than only one and of course when we multiply fractions the result is always smaller. This reflects the increasing difficulty of satisfying two conditions.

The Addition Rule

If *A* and *B* are defined on a sample space, then:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

We can think of the union symbol substituting for the word "or". The reason we subtract the intersection of *A* and *B* is to keep from double counting elements that are in both *A* and *B*.

Addition Rule for Mutually Exclusive Events

If *A* and *B* are **mutually exclusive**, then $P(A \cap B) = 0$. Therefore, for **mutually exclusive events**, the addition rule can be written as:

$$P(A \cup B) = P(A) + P(B)$$

Example 3-14

Klaus is trying to choose where to go on vacation. His two choices are: A = New Zealand and B = Alaska

- Klaus can only afford one vacation. The probability that he chooses A is P(A) = 0.6 and the probability that he chooses B is P(B) = 0.35.
- P(A∩B)=0 because Klaus can only afford to take one vacation

What is the probability that Klaus chooses to go to New Zealand or Alaska? What is the probability that he does not go anywhere on vacation?

Solution 3-14

The probability that he chooses either New Zealand or Alaska is $P(A \cup B) = P(A) + P(B) = 0.6 + 0.35 = 0.95$.

The probability that he does not choose to go anywhere on vacation is $1 - 0.95 = 0.05$.

Example 3-15

Carlos plays college soccer. He makes a goal 65% of the time he shoots. Carlos is going to attempt two goals in a row in the next game. A = the event Carlos is successful on his first attempt. P(A) = 0.65. B = the event Carlos is successful on his second attempt. P(B) = 0.65. Carlos tends to shoot in streaks. The probability that he makes the second goal | that he made the first goal is 0.90.

a) What is the probability that he makes both goals?

b) What is the probability that Carlos makes either the first goal or the second goal?

c) Are A and B independent?

d) Are A and B mutually exclusive?

Solution 3-15

a) The problem is asking you to find $P(A \cap B) = P(B \cap A)$. Since $P(B|A) = 0.90$: $P(B \cap A) = P(B|A)P(A) = (0.90)(0.65) = 0.585$

Carlos makes the first and second goals with probability 0.585.

b) The problem is asking you to find $P(A \cup B)$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.65 + 0.65 - 0.585 = 0.715$

Carlos makes either the first goal or the second goal with probability 0.715.

c) No, they are not, because $P(B \cap A) = 0.585$.

$P(B)P(A) = (0.65)(0.65) = 0.423$

$0.423 \neq 0.585 = P(B \cap A)$

So, $P(B \cap A)$ is not equal to $P(B)P(A)$.

d) No, they are not because $P(A \cap B) = 0.585$.

To be mutually exclusive, $P(A \cap B)$ must equal zero.

Try It 3-11

Helen plays basketball. For free throws, she makes the shot 75% of the time. Helen must now attempt two free throws. C = the event that Helen makes the first shot. P(C) = 0.75. D = the event Helen makes the second shot. P(D) = 0.75. The probability that Helen makes the second free throw given that she made the first is 0.85. What is the probability that Helen makes both free throws?

Chapter 3 Try It Solutions

Example 3-16

A community swim team has 150 members. Seventy-five of the members are advanced swimmers. Forty-seven of the members are intermediate swimmers. The remainder are novice swimmers. Forty of the advanced swimmers practice four times a week. Thirty of the intermediate swimmers practice four times a week. Ten of the novice swimmers practice four times a week. Suppose one member of the swim team is chosen randomly. What is the probability that the member is a novice swimmer?

a) What is the probability that the member practices four times a week?

b) What is the probability that the member is an advanced swimmer and practices four times a week?

c) What is the probability that a member is an advanced swimmer and an intermediate swimmer? Are being an advanced swimmer and an intermediate swimmer mutually exclusive? Why or why not?

d) Are being a novice swimmer and practicing four times a week independent events? Why or why not?

Solution 3-16

a) $\frac{28}{150}$

b) $\frac{80}{150}$

c) $\frac{40}{150}$

d) P(advanced ∩ intermediate) = 0, so these are mutually exclusive events. A swimmer cannot be an advanced swimmer and an intermediate swimmer at the same time.

e) No, these are not independent events.
P(novice ∩ practices four times per week) = 0.0667
P(novice)P(practices four times per week) = 0.0996
$0.0667 \neq 0.0996$

Try It 3-12

A school has 200 seniors of whom 140 will be going to college next year. Forty will be going directly to work. The remainder are taking a gap year. Fifty of the seniors going to college play sports. Thirty of the seniors going directly to work play sports. Five of the seniors taking a gap year play sports. What is the probability that a senior is taking a gap year?

Chapter 3 Try It Solutions

Example 3-17

Felicity attends Modesto JC in Modesto, CA. The probability that Felicity enrolls in a math class is 0.2 and the probability that she enrolls in a speech class is 0.65. The probability that she enrolls in a math class || that she enrolls in speech class is 0.25.

Let: M = math class, S = speech class, $M|S$ = math given speech

a) What is the probability that Felicity enrolls in math and speech?
   Find $P(M \cap S) = P(M|S)P(S)$.

b) What is the probability that Felicity enrolls in math or speech classes?
   Find $P(M \cup S) = P(M) + P(S) - P(M \cap S)$.

c) Are M and S independent? Is $P(M|S) = P(M)$?

d) Are M and S mutually exclusive? Is $P(M \cap S) = 0$?

Solution 3-17

a) 0.1625, b) 0.6875, c) No, d) No

Try It 3-13

A student goes to the library. Let events B = the student checks out a book and D = the student check out a DVD. Suppose that $P(B) = 0.40$, $P(D) = 0.30$ and $P(D|B) = 0.5$.

a) Find $P(B \cap D)$.

b) Find $P(B \cup D)$.

Chapter 3 Try It Solutions

Example 3-18

Studies show that about one woman in seven (approximately 14.3%) who live to be 90 will develop breast cancer. Suppose that of those women who develop breast cancer, a test is negative 2% of the time. Also suppose that in the general population of women, the test for breast cancer is negative about 85% of the time. Let B = woman develops breast cancer and let N = tests negative. Suppose one woman is selected at random.

a) What is the probability that the woman develops breast cancer? What is the probability that woman tests negative?

b) Given that the woman has breast cancer, what is the probability that she tests negative?

c) What is the probability that the woman has breast cancer AND tests negative?

d) What is the probability that the woman has breast cancer or tests negative?

e) Are having breast cancer and testing negative independent events?

Solution 3-18

a) $P(B) = 0.143$; $P(N) = 0.85$

b) $P(N \mid B) = 0.02$

c) $P(B \cap N) = P(B)P(N \mid B) = (0.143)(0.02) = 0.0029$

d) $P(B \cup N) = P(B) + P(N) - P(B \cap N) = 0.143 + 0.85 - 0.0029 = 0.9901$

e) No. $P(N) = 0.85$; $P(N \mid B) = 0.02$. So, $P(N \mid B)$ does not equal $P(N)$.

f) No. $P(B \cap N) = 0.0029$. For B and N to be mutually exclusive, $P(B \cap N)$ must be zero.

Try It 3-14

A school has 200 seniors of whom 140 will be going to college next year. Forty will be going directly to work. The remainder are taking a gap year. Fifty of the seniors going to college play sports. Thirty of the seniors going directly to work play sports. Five of the seniors taking a gap year play sports. What is the probability that a senior is going to college and plays sports?

Chapter 3 Try It Solutions

Example 3-19

Refer to the information in Example 3-18. P = tests positive.

a) Given that a woman develops breast cancer, what is the probability that she tests positive. Find $P(P|B) = 1 - P(N|B)$.

b) What is the probability that a woman develops breast cancer and tests positive. Find $P(B \cap P) = P(P|B)P(B)$.

c) What is the probability that a woman does not develop breast cancer. Find $P(B') = 1 - P(B)$.

d) What is the probability that a woman tests positive for breast cancer. Find $P(P) = 1 - P(N)$.

Solution 3-19

a) 0.98; b) 0.1401; c) 0.857; d) 0.15

Try It 3-15

A student goes to the library. Let events B = the student checks out a book and D = the student checks out a DVD. Suppose that $P(B) = 0.40$, $P(D) = 0.30$ and $P(D \mid B) = 0.5$.

a) Find $P(B')$.

b) Find $P(D \cap B)$.

c) Find $P(B \mid D)$.

d) Find $P(D \cap B')$.

e) Find $P(D \mid B')$.

Chapter 3 Try It Solutions

3.4 | Contingency Tables and Probability Trees

Contingency Tables

A **contingency table** provides a way of portraying data that can facilitate calculating probabilities. The table helps in determining conditional probabilities quite easily. The table displays sample values in relation to two different variables that may be dependent or contingent on one another. Later on, we will use contingency tables again, but in another manner.

Example 3-20

Suppose a study of speeding violations and drivers who use cell phones produced the following fictional data:

	Speeding violation in the last year	**No speeding violation in the last year**	**Total**
Uses cell phone while driving	25	280	305
Does not use cell phone while driving	45	405	450
Total	70	685	755

Table 3-2

The total number of people in the sample is 755. The row totals are 305 and 450. The column totals are 70 and 685. Notice that 305 + 450 = 755 and 70 + 685 = 755.

Calculate the following probabilities using the table.

a) Find P(Driver is a cell phone user).

b) Find P(Driver had no violation in the last year).

c) Find P(Driver had no violation in the last year ∩ was a cell phone user).

d) Find P(Driver is a cell phone user ∪ driver had no violation in the last year).

e) Find P(Driver is a cell phone user | driver had a violation in the last year).

f) Find P(Driver had no violation last year | driver was not a cell phone user)

Solution 3-20

a) $\frac{number\ of\ cell\ phone\ users}{total\ number\ in\ study} = \frac{305}{755}$

b) $\frac{number\ that\ had\ no\ violation}{total\ number\ in\ study} = \frac{685}{755}$

c) $\frac{280}{755}$

d) $\frac{305}{755}+\frac{685}{755}-\frac{280}{755}=\frac{710}{755}$

e) $\frac{25}{70}$ (The sample is reduced to the number of drivers who had a violation.)

f) $\frac{405}{450}$ (The sample space is reduced to the number of drivers who were not cell phone users.)

Try It 3-16

Table 3-3 shows the number of athletes who stretch before exercising and how many had injuries within the past year.

	Injury in last year	**No injury in last year**	**Total**
Stretches	55	295	350
Does not stretch	231	219	450
Total	286	514	800

Table 3-3

What is *P*(athlete stretches before exercising)?

What is *P*(athlete stretches before exercising||no injury in the last year)?

Chapter 3 Try It Solutions

Example 3-21

Table 3-4 shows a random sample of 100 hikers and the areas of hiking they prefer.

Sex	**The coastline**	**Near lakes and streams**	**On mountain peaks**	**Total**
Female	18	16	____	45
Male	____	____	14	55
Total	____	41	____	____

Table 3-4

a) Complete the table.

b) Are the events "being female" and "preferring the coastline" independent events?

Let F = being female and let C = preferring the coastline.

Find P(F∩C)P(F∩C).

Find P(F)P(C)

Are these two numbers the same? If they are, then F and C are independent. If they are not, then F and C are not independent.

c) Find the probability that a person is male given that the person prefers hiking near lakes and streams. Let M = being male, and let L = prefers hiking near lakes and streams.

   i. What word tells you this is a conditional?

   ii. Fill in the blanks and calculate the probability: P(___|___) = ___.

   iii. Is the sample space for this problem all 100 hikers? If not, what is it?

d) Find the probability that a person is female or prefers hiking on mountain peaks. Let F = being female, and let P = prefers mountain peaks.

   i. Find P(F).

   ii. Find P(P).

   iii. Find P(F∩P)P(F∩P).

   iv. Find P(F∪P)P(F∪P).

Solution 3-21

a)

Sex	**The coastline**	**Near lakes and streams**	**On mountain peaks**	**Total**
Female	18	16	**11**	45
Male	**16**	**25**	14	55

Total	**34**	41	**25**	100

b)

i. $P(F \cap C) = \frac{18}{100} = 0.18$

ii. $P(F)P(C) = \left(\frac{45}{100}\right)\left(\frac{34}{100}\right) = (0.45)(0.34) = 0.153$

P(F∩C)P(F∩C) ≠ P(F)P(C), so the events F and C are not independent.

c)

i. The word 'given' tells you that this is a conditional.

ii. $P(M|L) = \frac{25}{41}$

iii. No, the sample space for this problem is the 41 hikers who prefer lakes and streams.

d)

i. $P(F) = \frac{\mathbf{45}}{\mathbf{100}}$

ii. $P(P) = \frac{\mathbf{25}}{\mathbf{100}}$

iii. $P(F \cap P) = \frac{11}{100}$

iv. $P(F \cup P) = \frac{45}{100} + \frac{25}{100} - \frac{11}{100} = \frac{59}{100}$

Try It 3-17

Table 3-5 shows a random sample of 200 cyclists and the routes they prefer. Let *M* = males and *H* = hilly path.

Gender	**Lake path**	**Hilly path**	**Wooded path**	**Total**
Female	45	38	27	110
Male	26	52	12	90
Total	71	90	39	200

Table 3-5

Out of the males, what is the probability that the cyclist prefers a hilly path?

Are the events “being male” and “preferring the hilly path” independent events?

Chapter 3 Try It Solutions

Example 3-22

Muddy Mouse lives in a cage with three doors. If Muddy goes out the first door, the probability that he gets caught by Alissa the cat is $\frac{1}{5}$ and the probability he is not caught is $\frac{4}{5}$. If he goes out the second door, the probability he gets caught by Alissa is $\frac{1}{4}$ and the probability he is not caught is $\frac{3}{4}$. The probability that Alissa catches Muddy coming out of the third door is $\frac{1}{2}$ and the probability she does not catch Muddy is $\frac{1}{2}$. It is equally likely that Muddy will choose any of the three doors so the probability of choosing each door is $\frac{1}{3}$.

Caught or not	**Door one**	**Door two**	**Door three**	**Total**
Caught	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{6}$	_____
Not caught	$\frac{4}{15}$	$\frac{3}{12}$	$\frac{1}{6}$	_____
Total	_____	_____	_____	1

Table 3-6

- The first entry $\frac{1}{15} = \left(\frac{1}{5}\right)\left(\frac{1}{3}\right)$ is $P(Door\ One \cap Caught)$
- The entry $\frac{4}{15} = \left(\frac{4}{5}\right)\left(\frac{1}{3}\right)$ is $P(Door\ One \cap Not\ Caught)$

Verify the remaining entries.

a) Complete the probability contingency table. Calculate the entries for the totals. Verify that the lower-right corner entry is 1.

b) What is the probability that Alissa does not catch Muddy?

c) What is the probability that Muddy chooses Door One ∪ Door Two given that Muddy is caught by Alissa?

Solution 3-22

a)

Caught or not	**Door one**	**Door two**	**Door three**	**Total**
Caught	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{19}{60}$
Not caught	$\frac{4}{15}$	$\frac{3}{12}$	$\frac{1}{6}$	$\frac{41}{60}$
Total	$\frac{5}{15}$	$\frac{4}{12}$	$\frac{2}{6}$	1

b) $\frac{41}{60}$

c) $\frac{9}{19}$

Example 3-23

Table 3-7 contains the number of crimes per 100,000 inhabitants from 2008 to 2011 in the U.S.

Year	**Robbery**	**Burglary**	**Rape**	**Vehicle**	**Total**
2008	145.7	732.1	29.7	314.7	
2009	133.1	717.7	29.1	259.2	
2010	119.3	701	27.7	239.1	
2011	113.7	702.2	26.8	22	
Total					

Table 3-7 United States Crime Index Rates Per 100,000 Inhabitants 2008–2011

TOTAL each column and each row. Total data = 4,520.7

a) Find P(2009∩Robbery).

b) Find P(2010∩Burglary).

c) Find P(2010∪Burglary).

d) Find *P*(2011|Rape).

e) Find P(Vehicle|2008).

Solution 3-23

a) 0.0294, b) 0.1551, c) 0.7165, d) 0.2365, e) 0.2575

Try It 3-18

Table 3-8 relates the weights and heights of a group of individuals participating in an observational study.

Weight/height	**Tall**	**Medium**	**Short**	**Totals**
Obese	18	28	14	
Normal	20	51	28	
Underweight	12	25	9	
Totals				

Table 3-8

a) Find the total for each row and column

b) Find the probability that a randomly chosen individual from this group is Tall.

c) Find the probability that a randomly chosen individual from this group is Obese and Tall.

d) Find the probability that a randomly chosen individual from this group is Tall given that the individual is Obese.

e) Find the probability that a randomly chosen individual from this group is Obese given that the individual is Tall.

f) Find the probability a randomly chosen individual from this group is Tall and Underweight.

g) Are the events Obese and Tall independent?

Chapter 3 Try It Solutions

Tree Diagrams

Sometimes, when the probability problems are complex, it can be helpful to graph the situation. Tree diagrams can be used to visualize and solve conditional probabilities.

A **tree diagram** is a special type of graph used to determine the outcomes of an experiment. It consists of "branches" that are labeled with either frequencies or probabilities. Tree diagrams can make some probability problems easier to visualize and solve. The following example illustrates how to use a tree diagram.

Example 3-24

In an urn, there are 11 balls. Three balls are red (R) and eight balls are blue (B). Draw two balls, one at a time, **with replacement**. "With replacement" means that you put the first ball back in the urn before you select the second ball. The tree diagram using frequencies that show all the possible outcomes follows.

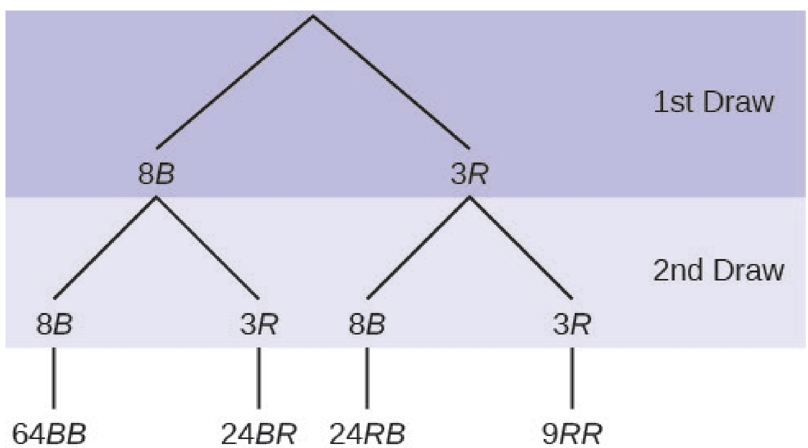


Figure 3.2 Total = 64 + 24 + 24 + 9 = 121

The first set of branches represents the first draw. The second set of branches represents the second draw. Each of the outcomes is distinct. In fact, we can list each red ball as $R1$, $R2$, and $R3$ and each blue ball as $B1$, $B2$, $B3$, $B4$, $B5$, $B6$, $B7$, and $B8$. Then the nine RR outcomes can be written as:

$R1R1$; $R1R2$; $R1R3$; $R2R1$; $R2R2$; $R2R3$; $R3R1$; $R3R2$; $R3R3$

The other outcomes are similar.

There are a total of 11 balls in the urn. Draw two balls, one at a time, with replacement. There are 11(11) = 121 outcomes, the size of the sample space.

a) List the 24 BR outcomes: B1R1, B1R2, B1R3, ...
b) Using the tree diagram, calculate P(RR).
c) Using the tree diagram, calculate P(RB ∪ BR).
d) Using the tree diagram, calculate P(R on 1st draw ∩ B on 2nd draw).
e) Using the tree diagram, calculate P(R on 2nd draw | B on 1st draw).
f) Using the tree diagram, calculate P(BB).
g) Using the tree diagram, calculate P(B on the 2nd draw | R on the first draw).

Solution 3-24

a) $B1R1$; $B1R2$; $B1R3$; $B2R1$; $B2R2$; $B2R3$; $B3R1$; $B3R2$; $B3R3$; $B4R1$; $B4R2$; $B4R3$; $B5R1$; $B5R2$; $B5R3$; $B6R1$; $B6R2$; $B6R3$; $B7R1$; $B7R2$; $B7R3$; $B8R1$; $B8R2$; $B8R3$

b) $P(RR) = \left(\frac{3}{11}\right)\left(\frac{3}{11}\right) = \frac{9}{121}$

c) $P(RB \cup BR) = \left(\frac{3}{11}\right)\left(\frac{8}{11}\right) + \left(\frac{8}{11}\right)\left(\frac{3}{11}\right) = \frac{48}{121}$

d) $P(R\ on\ 1st\ draw \cap B\ on\ 2nd\ draw) = \left(\frac{3}{11}\right)\left(\frac{8}{11}\right) = \frac{24}{121}$

e) $P(R\ on\ 2nd\ draw | B\ on\ 1st\ draw) = P(R\ on\ 2nd | B\ on\ 1st) = \frac{24}{88} = \frac{3}{11}$

This problem is a conditional one. The sample space has been reduced to those outcomes that already have a blue on the first draw. There are 24 + 64 = 88 possible outcomes (24 BR and 64 BB) Twenty-four of the 88 possible outcomes are BR. $\frac{24}{88} = \frac{3}{11}$.

f) $P(BB) = \frac{64}{121}$

g) $P(B\ on\ 2nd\ draw | R\ on\ 1st\ draw) = \frac{8}{11}$

There are 9 + 24 outcomes that have R on the first draw (9 RR and 24 RB). The sample space is then 9 + 24 = 33. 24 of the 33 outcomes have B on the second draw. The probability is then $\frac{24}{33} = \frac{8}{11}$.

Try It 3-19

In a standard deck, there are 52 cards. 12 cards are face cards (event *F*) and 40 cards are not face cards (event *N*). Draw two cards, one at a time, with replacement. All possible outcomes are shown in the tree diagram as frequencies. Using the tree diagram, calculate *P*(*FF*).

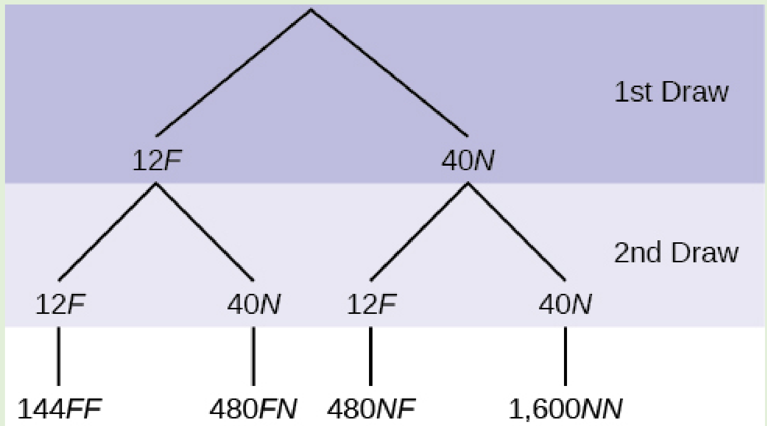


Figure 3.3

Chapter 3 Try It Solutions

Example 3-25

An urn has three red marbles and eight blue marbles in it. Draw two marbles, one at a time, this time without replacement, from the urn. **"Without replacement"** means that you do not put the first ball back before you select the second marble. Following is a tree diagram for this situation. The branches are labeled with probabilities instead of frequencies. The numbers at the ends of the branches are calculated by multiplying the numbers on the two corresponding branches, for example, $\left(\frac{3}{11}\right)\left(\frac{2}{10}\right) = \frac{6}{110}$.

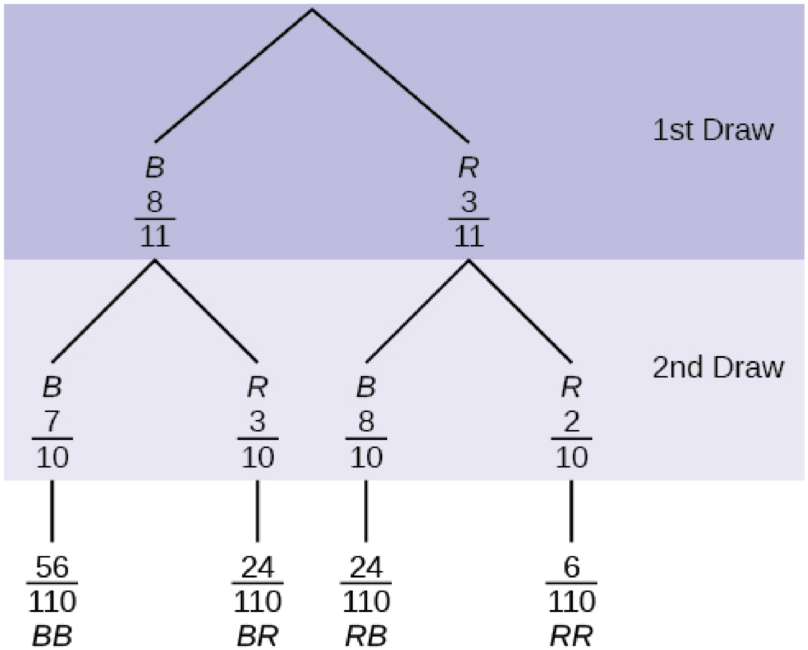


Figure 3.4 Total $= \frac{56+24+24+6}{110} = \frac{110}{110} = 1$

NOTE
If you draw a red on the first draw from the three red possibilities, there are two red marbles left to draw on the second draw. You do not put back or replace the first marble after you have drawn it. You draw **without replacement**, so that on the second draw there are ten marbles left in the urn.

Calculate the following probabilities using the tree diagram.
a) P(RR) = ________

b) Fill in the blanks:

$$P(RB \cup BR) = \left(\frac{3}{11}\right)\left(\frac{8}{10}\right) + (\_\_)(\_\_) = \frac{48}{110}$$

c) P(R on 2nd|B on 1st) =

d) Fill in the blanks.

$$P(R\ on\ 1st \cap B\ on\ 2nd) = (\_\_)(\_\_) = \frac{24}{100}$$

e. Find P(BB).

f. Find P(B on 2nd|R on 1st).

Solution 3-25

a) $P(RR) = \left(\frac{3}{11}\right)\left(\frac{2}{10}\right) = \frac{6}{110}$

b) $P(RB \cup BR) \; = \; \left(\frac{3}{11}\right)\left(\frac{8}{10}\right) \; + \; \left(\frac{8}{11}\right)\left(\frac{3}{10}\right) \; = \frac{48}{110}$

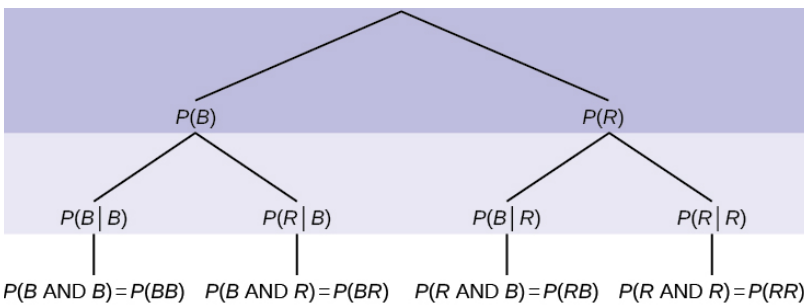
c) $P(R \; on \; 2nd | B \; on \; 1st) \; = \frac{3}{10}$

d) $P(R \; on \; 1st \cap B \; on \; 2nd) = \; \left(\frac{3}{11}\right)\left(\frac{8}{10}\right) \; = \frac{24}{100}$

e) $P(BB) = \left(\frac{8}{11}\right)\left(\frac{7}{10}\right)$

f) Using the tree diagram, $P(B \; on \; 2nd | R \; on \; 1st) = P(R|B) = \frac{8}{10}$.

If we are using probabilities, we can label the tree in the following general way.



- $P(R|R)$ here means $P(R$ on 2nd$|R$ on 1st$)$
- $P(B|R)$ here means $P(B$ on 2nd$|R$ on 1st$)$
- $P(R|B)$ here means $P(R$ on 2nd$|B$ on 1st$)$
- $P(B|B)$ here means $P(B$ on 2nd$|B$ on 1st$)$

Try It 3-20

In a standard deck, there are 52 cards. Twelve cards are face cards (*F*) and 40 cards are not face cards (*N*). Draw two cards, one at a time, without replacement. The tree diagram is labeled with all possible probabilities.

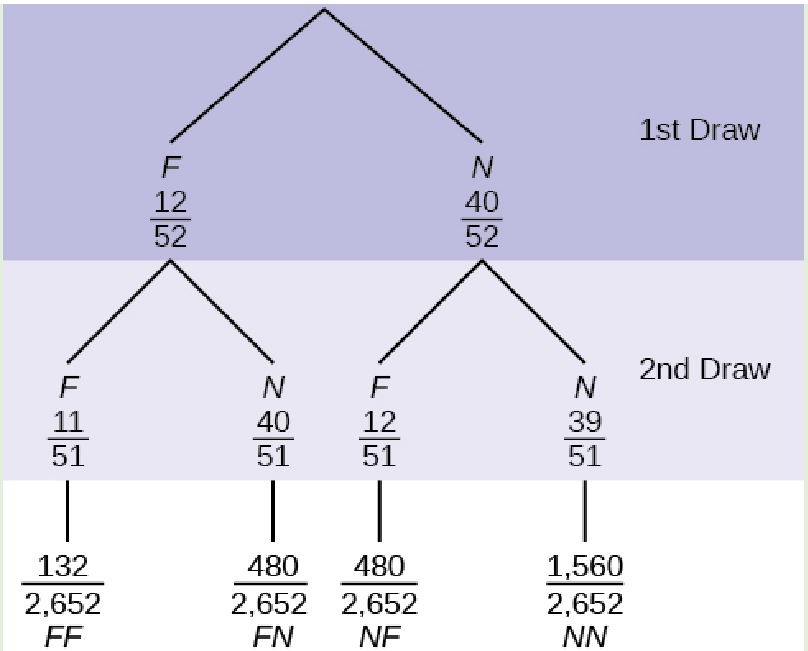


Figure 3.5

Find $P(FN \cup NF)$

Find $P(N|F)$.

Find $P(at\ most\ one\ face\ card)$.
Hint: "At most one face card" means zero or one face card.

Find $P(at\ least\ on\ face\ card)$.
Hint: "At least one face card" means one or two face cards.

Chapter 3 Try It Solutions

Example 3-26

A litter of kittens available for adoption at the Humane Society has four tabby kittens and five black kittens. A family comes in and randomly selects two kittens (without replacement) for adoption.

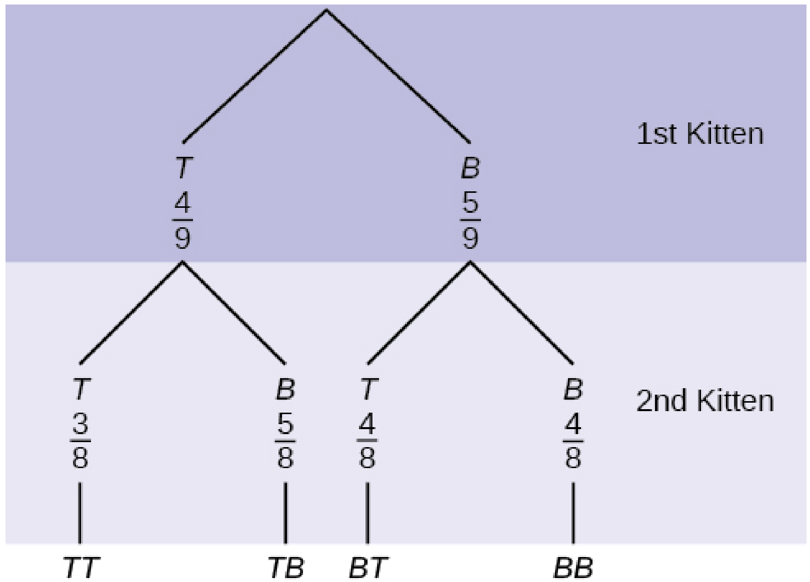


Figure 3.6

a) What is the probability that both kittens are tabby?

b) What is the probability that one kitten of each coloring is selected?

c) What is the probability that a tabby is chosen as the second kitten when a black kitten was chosen as the first?

d) What is the probability of choosing two kittens of the same color?

Solution 3-26

a) $\left(\frac{4}{9}\right)\left(\frac{3}{8}\right) = \frac{12}{72} = \frac{1}{6}$

b) $\left(\frac{4}{9}\right)\left(\frac{5}{8}\right) + \left(\frac{5}{9}\right)\left(\frac{4}{8}\right) = \frac{5}{9}$

c) $\frac{4}{8} = \frac{1}{2}$

d) $\frac{32}{72} = \frac{4}{9}$

Try It 3-21

Suppose there are four red balls and three yellow balls in a box. Two balls are drawn from the box without replacement. What is the probability that one ball of each coloring is selected?

Chapter 3 Try It Solutions

3.5 | Appendix: Principals of Counting

In an equally likely outcome experiment, computing probabilities means counting the number of all possible outcomes that make up the entire sample space as well as counting the number of outcomes in which an event may occur. In many experiments when listing all the possible outcomes or drawing the probability tree is cumbersome, the following counting rules may come handy.

Fundamental Counting Rule or the Multiplication Rule

For a sequence of two events A and B, in which the 1^{st} event A can occur in m ways and the 2^{nd} event B can occur in n ways, the events together can occur a total of $m \times n$ ways.

To use this counting rule, think of each choice as a slot, or position, to fill.

Example 3-27

An experiment consists of tossing a fair coin twice and recording the side that lands face-up on each toss. How many combinations of heads and tails is possible?

Solution 3-27

The number of combinations is

2 (number or choices for the 1^{st} toss) x 2 (number or choices for the 2^{nd} toss)= 4.

NOTATION:
- We write N(S)N(S) for the total number of outcomes (size of a sample space).

- N(A)N(A) denotes the number of outcomes in event AA.

Example 3-28

An experiment consists of tossing a fair coin once and then rolling a fair six-sided die and recording the sides that land face up. Find the size of the sample space N(S)N(S). Illustrate your answer by drawing a probability tree diagram.

Solution 3-28

The size of the sample space is $N(S)=2\times6=12$.

NOTE:
The fundamental principle of counting may be extended to any number of ordered independent events.

Example 3-29

A bank summer associate outfit consists of a pair of pants, a shirt, and a tie. Suppose he can choose from among four pairs of pants, six shirts, and three ties. How many different outfits are possible?

Solution 3-29

Using the counting principle, there are $N(S)=4\times6\times3=72$ outfits.

Example 3-30

(Traveling Salesmen problem) A franchise supervisor must visit eight different locations around the country. She can visit them in any order, but wishes to find the most convenient sequence. How many sequences are possible?

Solution 3-30

We think of each location as a slot that can be filled with a number of choices:

Number of choices available	8	7	6	5	4	3	2	1
Location	1st	2nd	3rd	4th	5th	6th	7th	8th

Then, by extended multiplication rule, the total number of sequences is

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8!$$

The notation (above read "eight factorial") is a shorthand for the product of all the integers from 1 to 8.

Factorial Notation

$$n! = 1 \times 2 \times 3 \times 4 \times \cdots \times (n-1) \times n$$

$$0! = 1 \qquad 1! = 1$$

Factorial Rule

A collection of n different items can be arranged in $n!$ different ways.

Example 3-31

In how many ways the letters of the word "MATH" can be arranged?

Solution 3-31

The word contains four different items. It can thus be arranged in $4!=24$ ways.

Note: Factorial calculations (for relatively small numbers) can be done using the factorial key on the calculator.

Permutations

The word *permutation* means an arrangement or a sequence and implies that the ORDER of items is IMPORTANT.

Example 3-32

Given three letters A B C, how many different arrangements are possible?

Solution 3-32

By the factorial rule there are 3!=6 ways to arrange three different letters:

ABC CAB BCA ACB CBA BAC,

forming six different sequences as long as ORDER matters.

Permutations Rule: (When items are all different)

The number of permutations (or sequences) of r items selected from n available items (not allowing repetition) is

$${}_nP_r = \frac{n!}{(n-r)!}$$

It is important to recognize that the permutations rule requires the following conditions:

- There are a total n different items available. (This rule does not apply if some of the items are identical to others.)
- We select r of the n items (without repetition).
- We consider rearrangements of the same items to be different sequences.

Example 3-33

In how many ways could a president and vice-president for a college club be chosen from a group of 30 students?

Solution 3-33

The number of different items available is n=30. The number of items to be selected is r=2. Note the order is important. So by the permutation rule, we calculate:

$$_{n}P_{r} = {}_{30}P_{2} = \frac{30!}{(30-2)!} = 870$$

Note, this calculation can be done using the respective button on the calculator.

Example 3-34

In a renting house with 13 students-roommates, three of the students are selected at random every week to clean up the common areas. The first one selected washes all the dishes, the second one cleans the bathroom, and the third one vacuums the living room. How many different "cleaning crews" are possible?

Solution 3-34

We compute $_{13}P_{3} = 1716$.

Permutations Rule: (When some items are identical to others)

If there are n items with n_1 alike, n_2 alike,$\cdots$, n_k alike, such that $n = n_1 + n_2 + \cdots + n_k$, then the number of permutations of all n items is:

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$

Example 3-35

In how many ways the letters of the word "STATISTICS" can be rearranged?

Solution 3-35

There are total of $n = 10$ items=letters, but some of them are identical: the letter “S” repeats 3 times, the letter “T” 3 times, letter “I” 2 times, and the letters “A” and “C” do not repeat (appear only once). Applying the formula yields:

$$\frac{10!}{3!\,3!\,2!\,1!\,1!} = 50{,}400$$

The Combination Formula

If the order of the selected items is NOT important, any selection is called a *combination*, that is, an unordered arrangement.

Recall in Example 3-32, the collection of three letters A, B, C provides for six permutations but only one combination.

The number of combinations of n items taken r at a time, is given by

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

It is important to recognize that in applying the combinations rule, the following conditions apply:

- There are a total n different items available.
- We select r of the n items (without repetition).
- We consider rearrangements of the same items to be the same grouping.

Example 3-36

The manufacturing process requires selecting 4 items from a batch of 50 for inspection. In how many ways it can be done?

Solution 3-36

In this case we compute:

$$_nC_r = {_{50}C_4} = \frac{50!}{(50-4)!\,4!} = \frac{50 \times 49 \times 48 \times 47}{24} = 230{,}300$$

Note that 50! is already a huge number in your calculator, but doing a simplification first yields a much easier calculation.

Example 3-37

How many sets of 12 questions for an online quiz can be chosen from a pool of 23 questions?

Solution 3-37

We compute:

$$_nC_r = {_{23}C_{12}} = \frac{23!}{(23-12)!\,12!} = 1{,}352{,}078$$

Using a Spreadsheet for Calculations

The common spreadsheets (LibreOffice (free), Microsoft Excel,..) offer common functions, including a permutation function with the syntax *PERMUT(n,r)* and combination function with the syntax *COMBIN(n,r)*. The built-in factorial function is used as *FACT(n)*.

Using a spreadsheet we obtain:

Counting Rule	**Excel Formula**	**Example**	**Excel Input**
Factorial	$= FACT(number)$	$8! = 40{,}320$	$= FACT(8)$
Permutation	$= PERMUT(number)$	${}_7P_3 = 210$	$= PERMUT(7{,}3)$
Combination	$= COMBIN(number)$	${}_{12}C_4 = 495$	$= COMBIN(12{,}4)$

Try It 3-22

How many blood types are possible in total if blood can be Rh+ or Rh- and belong to one of four blood groups A, B, AB, O?

Chapter 3 Try It Solutions

Try It 3-23

A marketing consultant typically tailors her sales scripts to the target audience. For these purposes the potential consumers are divided by gender (male, female), by age (into 5 groups) and by the level of household income (into 3 groups). How many different scripts are necessary to prepare?

Chapter 3 Try It Solutions

Try It 3-24

If a monkey typed four strokes on a keyboard and each stroke was a different letter of the alphabet, what is the probability that the monkey typed the word "MATH"?

Chapter 3 Try It Solutions

Try It 3-25

There are staff cutbacks in the department. There are five people eligible to stay on staff, but only three positions that can be maintained. In how many ways can the three positions be filled? Assume that the position ranks are different.

Chapter 3 Try It Solutions

Try It 3-26

a) Alice has 15 favorite songs to choose from. If she needs 7 songs in a specific order to choose for her half hour workout, how many different playlists are possible?

b) Sobia, Alice's friend, likes the same 15 songs and uses iTunes shuffle, which randomly selects songs from the list, to make her 7 song workout playlist. How many different playlists are possible for Sobia?

Chapter 3 Try It Solutions

Try It 3-27

A baby is given the letters O, T, O, T, N, R, O. Find the probability that he spells the word "TORONTO" on the first try.

Chapter 3 Try It Solutions

Try It 3-28

Assume that boys and girls are equally likely to be born for a specific couple.
a) Find the probability that a couple with three children will have exactly two boys.
b) Find the probability that a couple with 8 children will have exactly 5 boys and 3 girls.

Chapter 3 Try It Solutions

Try It 3-29

A telephone number consists of seven digits, the first three representing the exchange. How many different telephone numbers are possible within the 647 exchange?

Chapter 3 Try It Solutions

Try It 3-30

There are six vacancies in an office: four for technical support and two for marketing analysts. Find the number of ways of filling the vacancies if after the first round of interviews 15 people were selected for technical support positions, and 10 people were selected for marketing analysts' positions.

Chapter 3 Try It Solutions

Try It 3-31

The access code for a home security system consists of four digits from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. How many access codes are possible if
a) each digit can be used only once and not repeated?

b) each digit can be repeated?

c) each digit cannot be repeated and the first digit must be 1, 2, or 3?

d) each digit can be repeated but the first digit cannot be 1,2, or 3?

e) only a group of 4 numbers is required (rearrangements do not matter)?

Chapter 3 Try It Solutions

Chapter 3 Try It Solutions

Try It 3-1

a) With replacement

b) No

Try It 3-2

without replacement: a) Possible; b) Impossible; c) Possible

with replacement: a) Possible; b) Possible; c) Possible

Try It 3-3

The sample space of drawing two cards with replacement from a standard 52-card deck with respect to color is {*BB*, *BR*, *RB*, *RR*}.

Event *A* = Getting at least one black card = {*BB*, *BR*, *RB*}

$$P(A) = \frac{3}{4} = 0.75$$

Try It 3-4

a) $P(F) = \frac{1}{4}$

b) $P(G) = \frac{1}{2}$

c) $P(H) = \frac{1}{2}$

d) Yes

e) No

Try It 3-5

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.2} = 0.4 = P(A)$$

The events are independent because $P(A|B) = P(A)$.

Try It 3-6

Event *G* and *O* = {G1, G3}

$$P(G \cap O) = \frac{2}{10} = 0.2$$

Try It 3-7

a) (B|D)=0.6667

b) P(D|B)=0.5
c) No
d) No

Try It 3-8

P(B|A)=0.67

P(B)=0.25

So P(B)P(B) does not equal P(B|A)P(B|A) which means that B and A are not independent (wearing blue and rooting for the away team are not independent). They are also not mutually exclusive, because P(B∩A)=0.20, not 0.

Try It 3-9

Because P(I∩F)=0,

P(I∪F)=P(I)+P(F)−P(I∩F)=0.44+0.56−0=1

Try It 3-10

a) $P(T) = \frac{1}{4}$
b) $P(T|F) = \frac{1}{2}$
c) No
d) No
e) Yes

Try It 3-11

P(D|C) = 0.85

P(C ∩ D) = P(D ∩ C)
P(D ∩ C) = P(D|C)P(C) = (0.85)(0.75) = 0.6375
Helen makes the first and second free throws with probability 0.6375.

Try It 3-12

$P = \frac{200-140-40}{200} = \frac{20}{200} = 0.1$

Try It 3-13

a) P(B ∩ D) = P(D||B)P(B) = (0.5)(0.4) = 0.20.

b) P(B ∪ D) = P(B) + P(D) − P(B ∩ D) = 0.40 + 0.30 − 0.20 = 0.50

Try It 3-14

Let A = student is a senior going to college.

Let B = student plays sports.

$P(B) = \frac{140}{200}$

$P(B|A) = \frac{50}{140}$

$P(A \cap B) = P(B|A)P(A)$

$P(A \cap B) = \left(\frac{140}{200}\right)\left(\frac{50}{140}\right) = \frac{1}{4}$

Try It 3-15

a) $P(B') = 0.60$

b) $P(D \cap B) = P(D|B)P(B) = 0.20$

c) $P(B|D) = \frac{\boldsymbol{P(B \cap D)}}{\boldsymbol{P(D)}} = \frac{\mathbf{0.20}}{\mathbf{0.30}} = 0.66$

d) $P(D \cap B') = P(D) - P(D \cap B) = 0.30 - 0.20 = 0.10$

e) $P(D|B') = P(D \cap B')P(B') = (P(D) - P(D \cap B))(0.60) =$ $(0.10)(0.60) = 0.06$

Try It 3-16

a) $P(athlete\ stretches\ before\ exercising) = \frac{350}{800} = 0.4375$

b) $P(athlete\ stretches\ before\ exercising|no\ injury\ in\ the\ last\ year) = \frac{295}{514} =$ 0.5739

Try It 3-17

a) $\mathrm{P(H|M)} = \frac{52}{90} = 0.5778$

b) For M and H to be independent, show P(H|M) = P(H)

$$\mathrm{P(H|M)} = 0.5778,\ \mathrm{P(H)} = \frac{90}{200} = 0.45$$

P(H|M) does not equal P(H) so M and H are NOT independent.

Try It 3-18

Weight/height	**Tall**	**Medium**	**Short**	**Totals**
Obese	18	28	14	60
Normal	20	51	28	99
Underweight	12	25	9	46
Totals	50	104	51	205

a) Row Totals: 60, 99, 46. Column totals: 50, 104, 51.

b) $P(Tall) = \frac{50}{205} = \ 0.244$

c) $P(Obese \cap Tall) = \frac{18}{205} = \ 0.088$

d) $P(Tall||Obese) \ = \frac{18}{60} = 0.3$

e) $P(Obese|Tall) \ = \frac{18}{50} = \ 0.36$

f) $P(Tall \cap Underweight) \ = \frac{12}{205} = \ 0.0585$

g) No. *P*(Tall) does not equal *P*(Tall|Obese).

Try It 3-19

Total number of outcomes is 144 + 480 + 480 + 1600 = 2,704.

$$P(FF) = \frac{144}{144 + 480 + 480 + 1600} = \frac{144}{2704} = \frac{9}{169}$$

Try It 3-20

a) $P(FN \cup NF) = \frac{480}{2652} + \frac{480}{2652} = \frac{960}{2652} = \frac{80}{221}$

b) $P(N|F) = \frac{40}{51}$

c) $P(at\ most\ one\ face\ card) = \frac{480+480+1560}{2652} = \frac{2520}{2652}$

d) $P(at\ least\ one\ face\ card) = \frac{132+480+480}{2652} = \frac{1092}{2652}$

Try It 3-21

$\left(\frac{4}{7}\right)\left(\frac{3}{6}\right) + \left(\frac{3}{7}\right)\left(\frac{4}{6}\right) = \frac{4}{7}$

Try It 3-22

8

Try It 3-23

30

Try It 3-24

$\frac{1}{358{,}800}$

Try It 3-25

${}_5P_3 = 60$

Try It 3-26

a) 32,432,400
b) 6,435

Try It 3-27

$\frac{1}{420}$

Try It 3-28

a) $\frac{3}{8}$

b) $\frac{56}{256}$

Try It 3-29

$10^4 = 10{,}000$

Try It 3-30

${}_{15}C_4 \times {}_{10}C_2 = 61{,}425$

Try It 3-31

a) $10 \times 9 \times 8 \times 7 = 5{,}040$
b) $10^4 = 10{,}000$
c) $3 \times 9 \times 8 \times 7 = 1{,}512$
d) $7 \times 10 \times 10 \times 10 = 7{,}000$
e) ${}_{10}C_4 = 210$

KEY TERMS

Conditional Probability
the likelihood that an event will occur given that another event has already occurred

Equally Likely
Each outcome of an experiment has the same probability.

Event
a subset of the set of all outcomes of an experiment; the set of all outcomes of an experiment is called a **sample space** and is usually denoted by *S*. An event is an arbitrary subset in *S*. It can contain one outcome, two outcomes, no outcomes (empty subset), the entire sample space, and the like. Standard notations for events are capital letters such as *A*, *B*, *C*, and so on.

Experiment
a planned activity carried out under controlled conditions

Outcome
a particular result of an experiment

Probability
a number between zero and one, inclusive, that gives the likelihood that a specific event will occur; the foundation of statistics is given by the following 3 axioms (by A.N. Kolmogorov, 1930's): Let *S* denote the sample space and *A* and *B* are two events in *S*. Then:

- $0 \leq P(A) \leq 1$
- If *A* and *B* are any two mutually exclusive events, then $P(A \cup B) = P(A) + P(\text{B})$.
- $P(S) = 1$

Sample Space
the set of all possible outcomes of an experiment

The Intersection: the ∩ Event
An outcome is in the event $A \cap B$ if the outcome is in both $A \cap B$ at the same time.

The Complement Event
The complement of event *A* consists of all outcomes that are NOT in *A*.

The Conditional Probability of *A* | *B*
$P(A|B)$ is the probability that event *A* will occur given that the event *B* has already occurred.

The Union: the ∪ Event
An outcome is in the event $A \cup B$ if the outcome is in *A* or is in *B* or is in both *A* and *B*.

Dependent Events

If two events are NOT independent, then we say that they are dependent.

Sampling with Replacement

If each member of a population is replaced after it is picked, then that member has the possibility of being chosen more than once.

Sampling without Replacement

When sampling is done without replacement, each member of a population may be chosen only once.

Independent Events

The occurrence of one event has no effect on the probability of the occurrence of another event. Events A and B are independent if one of the following is true:

1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$
3. $P(A \cap B) = P(A)P(B)$

Mutually Exclusive

Two events are mutually exclusive if the probability that they both happen at the same time is zero. If events A and B are mutually exclusive, then $P(A \cap B) = 0$.

Tree Diagram

the useful visual representation of a sample space and events in the form of a “tree” with branches marked by possible outcomes together with associated probabilities (frequencies, relative frequencies)

Contingency Table

the method of displaying a frequency distribution as a table with rows and columns to show how two variables may be dependent (contingent) upon each other; the table provides an easy way to calculate conditional probabilities.

CHAPTER REVIEW

3.1 | Terminology

In this module we learned the basic terminology of probability. The set of all possible outcomes of an experiment is called the sample space. Events are subsets of the sample space, and they are assigned a probability that is a number between zero and one, inclusive.

3.2 | Independent and Mutually Exclusive Events

Two events *A* and *B* are independent if the knowledge that one occurred does not affect the chance the other occurs. If two events are not independent, then we say that they are dependent.

In sampling with replacement, each member of a population is replaced after it is picked, so that member has the possibility of being chosen more than once, and the events are considered to be independent. In sampling without replacement, each member of a population may be chosen only once, and the events are considered not to be independent. When events do not share outcomes, they are mutually exclusive of each other.

3.3 | Two Basic Rules of Probability

The multiplication rule and the addition rule are used for computing the probability of *A* and *B*, as well as the probability of *A* or *B* for two given events *A*, *B* defined on the sample space. In sampling with replacement each member of a population is replaced after it is picked, so that member has the possibility of being chosen more than once, and the events are considered to be independent. In sampling without replacement, each member of a population may be chosen only once, and the events are considered to be not independent. The events *A* and *B* are mutually exclusive events when they do not have any outcomes in common.

3.4 | Contingency Tables and Probability Trees

There are several tools you can use to help organize and sort data when calculating probabilities. Contingency tables help display data and are particularly useful when calculating probabilites that have multiple dependent variables.

A tree diagram use branches to show the different outcomes of experiments and makes complex probability questions easy to visualize.

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